Name________________________________ **Student ID #** _________________________
Instructor___________________________ **Lab Period** _____ **Date Due** ___________

Lab 10
Logarithmic Functions

Objectives

1. To develop an ability to visualize the graphs of logarithmic functions.
2. To become familiar with applications of the logarithmic function.

Data Table Operations

To select a column of the data table, move the mouse until the cursor is between the column label and the data table. The cursor will change its shape to a small downward-pointing arrow: ▼. Click the mouse to highlight (select) the column.

To use a data table command, scroll the command list to find the command that you want and double-click the command.

Exploration 1 *Carbon-14 Dating in Archaeology*

In 1949, J. R. Arnold and W. F. Libby published a paper describing the technique of radiocarbon dating of organic samples. Since that time, archaeologists all over the world have used radiocarbon dating to estimate the ages of the artifacts and fossils that they have discovered in their field work. The basis of radiocarbon dating is that cosmic radiation produces neutrons that react with the nitrogen in the atmosphere to produce a radioactive carbon isotope called carbon-14 (written as ${}^{14}C$). Plants absorb radioactive carbon-14 through carbon dioxide and animals (including humans) ingest ${}^{14}C$ by eating plants or other animals. However, when a plant or animal dies it stops replacing its carbon. After death, the amount of its ${}^{14}C$ decays at a known rate according to the natural law of exponential decay. It was estimated by Libby that the half-life of ${}^{14}C$ (the time it takes for the amount of ${}^{14}C$ to decrease by one half) is 5,568 years — although modern research has shown the half-life to be 5,730 years. Living organisms contain two types of carbon, radioactive ${}^{14}C$ and stable carbon ${}^{12}C$. Since the ratio of the amount of ${}^{14}C$ to the amount of ${}^{12}C$ is approximately constant in all living organisms, scientists can use the amount of ${}^{12}C$ found in an ancient object to estimate the original amount of ${}^{14}C$ in the ancient object, and thus, estimate the proportion of ${}^{14}C$ that has decayed. So by determining this proportion of ${}^{14}C$ that has decayed, it is possible to estimate the age of the object. The amount of ${}^{14}C$ in an object can be measured by a Geiger counter, special gas counters, or accelerator mass spectrometry.

1. Use the Exponential Decay model (law of natural decay) to find the equation that is used for carbon-14 dating:

$$x = \left(\frac{1}{2}\right)^{\frac{t}{5730}}$$

where x is the proportion of ^{14}C remaining in the object after t years. Show all your work below.

2. Solve the carbon-14 dating equation for t to get the Age function:

$$t = age(x) = \frac{5730\ln(x)}{\ln(1/2)}$$

where x is the proportion of ^{14}C remaining in the object after t years and t is the age of the object. Show all your work below.

Next, let's enter and plot the age function by doing the following:

- Select **New Function** from the **Work** menu and enter the age function **5730 ln(x)/ln(1/2)** into the **Work** window.
- Click in the **Domain & Range** window and set the plotting domain to $\mathbf{0 \le x \le 1}$.

- Select **Plot** from the **Graph** menu.

3. Using the graph of the age function, give a description of how the age of an ancient artifact increases as the proportion of its ^{14}C decreases (for example, what happens to the age function as the proportion of ^{14}C becomes small?). ………………………………

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Since it can be difficult to accurately read the value of the age from the graph, let's generate an age table for determining the ages of ancient artifacts. To do this, we will need a table that contains two columns with 100 entries each. The first column will contain the values for the proportion of ^{14}C remaining in the artifact and the second column will contain the estimated age of the artifact. Furthermore, we want the first column to contain the proportions 0.01, 0.02, 0.03, . . . , 1.00. To generate this table using *TEMATH*, follow these instructions:

- Select **New Data Table** from the **Work** menu.
- Double-click the **Resize** command. Enter **100** and press the **Enter** key.
- Select the first column of the data table. Double-click the **Fill** command. Enter the fill-function **0.01 i** (where i represents the row number of the table) and press the **Enter** key.
- Select the first column of the data table. Double-click the **Generate** command. Enter the generate-function **5730 ln(x)/ln(1/2)** and press the **Enter** key. This will fill the second column with values of the age function.
- Select **Print Data Table** from the **File** menu. This will print a copy of the Age table.

<u>Attach a copy of the age table to this report.</u>

Using the Age table, answer the following questions.

4. What is the age of an ancient artifact that contains 75% of its ^{14}C ? ……………………..

5. Sometimes the amount of ^{14}C is measured inaccurately due to such factors as background cosmic radiation and counting errors. Suppose it is only possible to measure the amount of ^{14}C to an accuracy of $\pm 3\%$. How accurately can one determine the age of the ancient artifact if the true percentage of ^{14}C remaining is somewhere between 72% and 78% (give the range of ages)? …………………………………………………………………

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6. What is the age of an ancient artifact that contains 25% of its ^{14}C? ……………………

7. a) What if the amount of ^{14}C was measured incorrectly. Suppose the correct amount was 23% instead. What is the revised age of the ancient artifact? ……………………

   b) By how much did the age change? ……………………………………………………

8. Describe how the estimate of the age of the artifact depends on the accuracy of the measurement of the ^{14}C. In particular, for what range of values of *x* does *the age* vary the most, and for what range of values of *x* should the amount of ^{14}C be measured most accurately? ……………………………………………………………………………

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9. What if we use the estimated half-life value of 5568 years that Libby used instead of the modern day value of 5730 years. **Note:** You will need to generate a new age table.

   a) What would now be the estimated age of the ancient artifact that contains 25% of its ^{14}C?……………………………………………………………………………

   b) By how much did the age change? ……………………………………………………

Exploration 2 *Short-Term Memory*

Psychologists have been studying learning, memory, and forgetting for many years. In 1962, Hellyer did a study on short-term memory. In his study, he had students memorize three-consonant nonsense syllables (for example, “btc” or “mnd”). The student repeated the nonsense syllable once and was then asked to recall the nonsense syllable after a period of 3, 9, 18, or 27 seconds. This period of time is referred to as the “Retention Interval.” Random digits were read to the student during the interval to recall. This was repeated many times and the mean percent of correct recalls was tabulated. In a typical experiment, the following data was obtained:

Retention Interval (sec.)	3	9	18	27
Mean % Correct	87	40	23	16

Let's find a mathematical model that represents this data. First, let's enter and plot the data by doing the following:

- If the Data Table window is not open, then select **New Data Table** from the **Work** menu, otherwise, click in the Data Table window to make it active.
- Resize the data table to four rows and enter the experimental data. Enter the retention interval data into the *x*-column and the mean % correct data into the *y*-column.
- Add the data table to the Work window by closing the **Data Table** window and clicking the **Yes** button in the **Add Data Table** dialog.
- Click *off* **autoscaling**, set the domain to $\mathbf{0 \le x \le 30}$ and set the range to $\mathbf{0 \le y \le 100}$.
- Plot the data.

There is a mathematical procedure called “The Method of Least Squares” that is used to *fit* mathematical functions to experimental data. In this exploration, we want to fit a function of the form $y(x) = a + b \ln(x)$ to the data. To do this in *TEMATH*, select **Least Squares — Logarithmic** from the **Tools** menu (be sure that the data table is selected in the Work window). The fitted logarithmic function will be written into the first cell of the Work window and its graph will be overlaid on top of the plot of the data.

1. a) What is the equation of the fitted logarithmic function as shown in the first row of the Work window? ……………………………………………………………………

   b) Suppose we want to use this fitted logarithmic function to model (describe) the learning process. Does it matter that the graph of this function does not pass through the data points exactly? ………. Explain. ……………………………………………

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c) Does this logarithmic model appear to be a good representation of the data?……….

Explain..………………………………………………………………………………

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In another part of this experiment, the student repeated the nonsense syllable *eight* times and was then asked to recall the nonsense syllable after a period of 3, 9, 18 or 27 seconds. For this part of the experiment, the following data was obtained:

Retention Interval (sec.)	3	9	18	27
Mean % Correct	99	89	76	71

Enter this data into the Data Table window, **Add** the data to the Work window, and overlay a plot of the data on top of the existing plot. Find the Least Squares logarithmic fit for this new data by selecting **Least Squares — Logarithmic** from the **Tools** menu.

2. a) What is the equation of the fitted logarithmic function? ………………………………

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b) Does this logarithmic model appear to be a good representation of the data? ....

Explain. ………………………………………………………………………………

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3. Considering the results from both parts of the experiment, does repeating the nonsense syllable more than once improve short-term memory? .... Discuss how the graphs of the fitted logarithmic models support your answer. ……………………………………

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Exploration 3 *A Learning Experiment*

A cat was placed in a box and was timed until it escaped. This was repeated many times to see if learning would take place and if the cat would escape in a shorter amount of time the more times it was placed in the box. The following data was obtained in a typical experiment:

Number of Trials	1	3	5	7	9	11	13	15	17
Seconds to Escape	100	60	40	40	25	30	25	25	20
Number of Trials	19	21	23	25					
Seconds to Escape	18	15	10	10					

1. Use the methods described in exploration 2 to answer the following questions:

   a) What is the equation of the fitted logarithmic function? ………………………………

   …………………………………………………………………………………………

b) Describe the learning process represented by this logarithmic function (for example, you might consider the following: For what number of trials does the most learning take place? For what number of trials does the least amount of learning take place? If each trial cost an amount of money to perform, what would be the most cost effective number of trials to perform to obtain the most learning?). …………………

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