Name_____ **Student ID #** _____

Instructor_____ **Lab Period** _____ **Date Due** _____

Lab 12
Parametric Equations

Objectives

1. To investigate applications of parametric equations.
2. To develop an understanding of how parametric equations can be used to solve problems.

To Select Parametric Plot Mode:

- Select **Parametric** from the **Graph** menu.

To Plot Parametric Equations:

- If you are not in Parametric Plot mode, select **Parametric** from the **Graph** menu.
- Select **New Function** from the **Work** menu. The function template **xy1(t) = [,]** appears in the first row of the **Work** window. Enter the x-coordinate function of t to the left of the comma and enter the y-coordinate function of t to the right of the comma. Click the **Enter** button or press the Enter or Return key. For example, enter the pair of parametric equations $x(t) = t + 2$ and $y(t) = t^2 + 3$ as

  xy1(t) = [t+2, t^2+3]

- Enter the domain for t and the range for x and y if you are not using autoscaling.
- Select **Plot** from the **Graph** menu.

Exploration *Writing Your Name*

The objective of this exploration is to draw your name (or someone else's name) using parametric curves. The name you select to draw must contain **at least three letters** and **at least one of the letters must contain a curved portion** (for example, B, C, D, G, J, O, P, Q, R, S, U). Credit will be given for styling, number of letters, and complexity of letters.

To get started, plot the x- and y-axes and the first quadrant by doing the following:

- Click *off* **Autoscaling** in the **Domain & Range** window.
- Set the *t* domain to $0 \le t \le 1$.
- Set the *x* range to $0 \le x \le 5$.
- Set the *y* range to $0 \le y \le 5$.
- Select **Plot Axes** from the **Graph** menu. The axes will be plotted in the **Graph** window.

Helpful Hint 1: Parameterization of a Line

A line between two points (x_1, y_1) and (x_2, y_2) can be parameterized using the equations derived below.

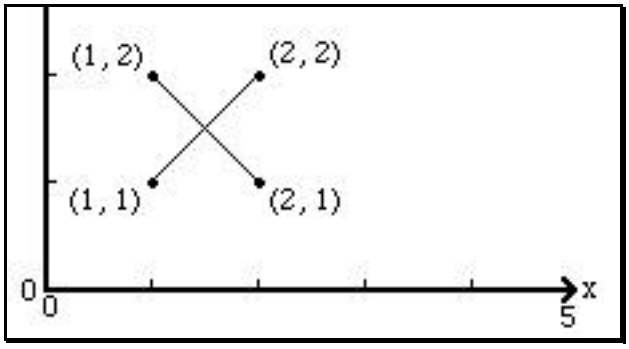
If we assume that *t* varies between 0 and 1, that is, $0 \le t \le 1$, then when $t = 0$ we want $x = x_1$ and when $t = 1$ we want $x = x_2$. Thus, we want to find the equation of the line between the two points $(t, x) = (0, x_1)$ and $(t, x) = (1, x_2)$. Using the point-slope form of the equation of a line, we get the equation $x - x_1 = \left(\frac{x_2 - x_1}{1 - 0}\right)(t - 0)$. Solving this equation for *x*, we obtain $x = x_1 + (x_2 - x_1)t$. Similarly, we can find the equation for *y* as $y = y_1 + (y_2 - y_1)t$.

As an example, to draw the letter X shown below using the two sets of points (1, 2), (2, 1) and (1, 1), (2, 2), you can use the following two pairs of parametric equations:

$$x(t) = 1 + 1t, \quad y(t) = 2 + (-1)t$$

and

$$x(t) = 1 + 1t, \quad y(t) = 1 + 1t$$



Helpful Hint 2: Parameterization of a Circle (Ellipse)

The unit circle can be parameterized by means of the equations

$$x = \cos(t), \quad y = \sin(t)$$

A full circle is drawn for the parameter interval $0 \le t \le 2\pi$. If the parameter interval is $0 \le t \le 1$, only a portion of the circle will be drawn. To draw a larger portion of the circle, select $a > 1$ and use the parametric equations

$$x = \cos(at), \quad y = \sin(at)$$

To move the center of the circle to (h, k), use the parametric equations

$$x = h + \cos(at), \quad y = k + \sin(at)$$

To change the radius of the circle, select an amplitude b and to change the shape of the circle to be elliptical use different amplitudes b and d where $b \neq d$ and use the parametric equations

$$x = h + b\cos(at), \quad y = k + d\sin(at)$$

To change the point where the circle begins to be drawn, add a phase shift ϕ to the trigonometric functions and use the parametric equations

$$x = h + b\cos(\phi + at), \quad y = k + d\sin(\phi + at)$$

Graphing Strategies:

Obviously, you will not get the correct parametric equations the first time you try so here are some tips on plotting the letters and erasing.

- Select **Plot** from the **Graph** menu to erase the current graph and replace it with the axes and a plot of the **selected** pair of parametric equations in the **Work** window. The **selected** pair of parametric equations have a box around them.
- Select any pair of parametric equations in the **Work** window by clicking on them and then select **Overlay Plot** from the **Graph** menu to plot the selected pair of parametric equations onto the plot in the **Graph** window.
- To plot or overlay the plot of a pair of parametric equations, they must be active, that is, a check mark must be visible to the left of the selected pair of parametric equations in the **Work** window. If a pair of parametric equations is not active, simply click on the position of the "check mark" to the left of the selected pair of parametric equations and the check mark will be made visible. Note that only five pairs of parametric equations can be active at one time. When you make a new pair of parametric equations active, an older pair will be made inactive.

Attach a printed copy of the **Graph** window containing your name to this lab report. Also, attach a printed copy of the **Work** window containing your parametric equations.