Zeroes of polynomials:

$f(x) = x^8 - 4x^7$

$f(x) = 0 \qquad x^8 - 4x^7 = 0$

$x^7(x-4) = 0$

$x^7 = 0 \quad$ or $\quad x-4=0$

Solutions $\quad x=0 \quad$ and $\quad x=4$

$x^5 - 8x^4 + x^3 = 0$

$x^3(x^2 - 8x + 1) = 0$

$x^3 = 0 \qquad x^2 - 8x + 1 = 0$

$x = 0 \qquad x = \frac{8 \pm \sqrt{64-4}}{2}$

$= 4 \pm \frac{\sqrt{60}}{2}$

$= 4 \pm \frac{\sqrt{4}\sqrt{15}}{\cancel{2}}$

$= 4 \pm \sqrt{15}$

$x^4 + 12x^2 - 3 = 0 \qquad y = x^2$

$y^2 + 12y - 3 = 0 \qquad y = \frac{-12 \pm \sqrt{144+12}}{2}$

$y = -6 \pm \sqrt{39}$

$x^2 = -6 \pm \sqrt{39} \qquad x = \pm\sqrt{\sqrt{39} - 6}$

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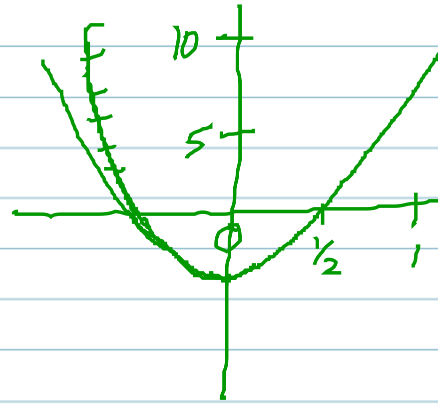
$x^2 = -6 \pm \sqrt{39} \qquad x = \pm\sqrt{\sqrt{39} - 6}$

$f(x) = x^4 + 12x^2 - 3$



$p(x) = x^2 - (\sqrt{39} - 6)^2$

$f(x) = 0$ has solutions at

$x = \pm .49473 \approx \pm\left(\sqrt{\sqrt{39} - 6}\right)$

Theorem: If $p(x)$ is a polynomial of degree n, then the eqn. $p(x) = 0$ has at most n real solutions. (This eqn. has exactly n complex solutions.)