Rational functions: Limit notation & horizontal asymptotes

For $f(x) = \cancel{\dfrac{x^2-3x+4}{x^2-5x+6}} = \dfrac{x^2-3x+4}{x^2-5x-6}$

As x gets closer to -1, but stays bigger than -1, $f(x)$ gets ~~bigger and bigger~~ more and more negative

i) As $x \to -1^+$, $f(x) \to -\infty$

ii) As $x \to -1^-$, $f(x) \to \infty$

i) $\lim_{x \to -1^+} f(x) = -\infty$ ii) $\lim_{x \to -1^-} f(x) = \infty$

$\lim_{x \to 6^+} f(x) = \infty$ and $\lim_{x \to 6^-} f(x) = -\infty$

As $x \to \infty$, $x^2-3x+4 \to \infty$

" , $x^2-5x+6 \to \infty$

$f(10) = \dfrac{100-30+4}{100-50+6} = \dfrac{74}{56}$

$f(100) = \dfrac{10{,}000-300+4}{10{,}000-500+6} = \dfrac{9{,}704}{9{,}506} = 1 + \ldots$

$f(1000000) = \dfrac{1{,}000{,}000{,}000{,}000 - 3{,}000{,}000 + 4}{1{,}000{,}000{,}000{,}000 - 5{,}000{,}000 + 6} = 1 +$ a little bit

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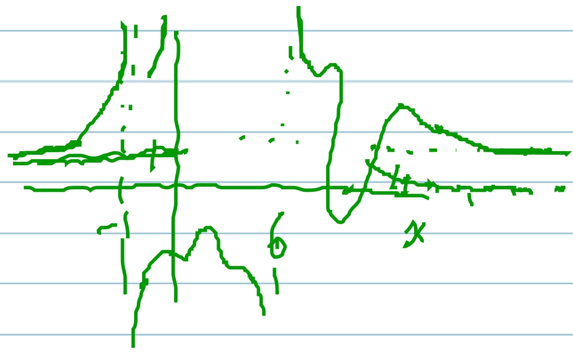
As $x \to \infty$, $f(x) \to 1$

$$f(x) = \frac{1 - 3/x + 4/x^2}{1 - 5/x + 6/x^2}$$

$\frac{4}{x^2} \to 0$ as $x \to \infty$

$\frac{3}{x} \to 0$ as $x \to \infty$

$f(x) \to 1$ as $x \to \infty$



$y = x$ is a horizontal asymptote of f

As $x \to -\infty$, $f(x) \to 1$

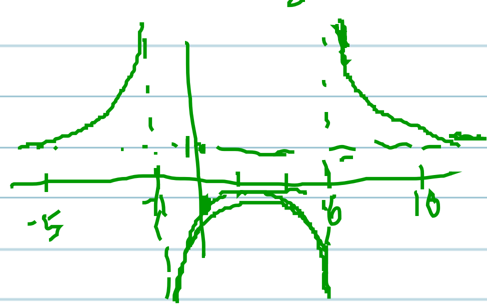
$f(x) = 0$ if $x^2 - 3x + 4 = 0$ at some x for which $x^2 - 5x + 6 \neq 0$

$x^2 - 3x + 4 = 0 = \cancel{(x-4)(x+1)}$

~~$x = 4$ or $x = -1$~~

$$x = \frac{3 \pm \sqrt{9 - 16}}{2}$$

$\sqrt{9-16} = \sqrt{-7}$ which does not exist!



$f(0) = -4/3$