

Rational Functions



$$f(x) = \frac{x^2 - 3x + 4}{x^2 - 5x - 6}$$

Rational function: The quotient of two polynomials

$$f(x) = \frac{p(x)}{q(x)}$$

f is undefined at the zeroes of q

Domain of f is all real numbers except the solutions of $q(x) = 0$

Domain of the above f :

$$x^2 - 5x - 6 = 0$$

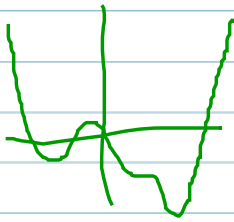
$$(x-6)(x+1) = 0$$

$$x-6 = 0 \quad \text{or} \quad x+1 = 0$$

$$x=6 \quad \text{or} \quad x=-1$$

Domain of f is all real numbers except 6 and -1
or $(-\infty, -1) \cup (-1, 6) \cup (6, \infty)$

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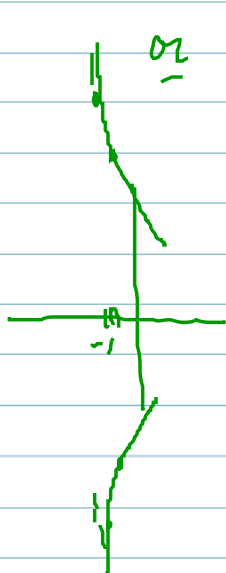
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$$f(-1 + \text{a little bit}) = \frac{(-1 + 1.6)^2 - 3(-1 + 1.6) + 4}{0 \pm \text{some other little bit}}$$

$$= \frac{8 \pm \text{yet another little bit}}{\pm \text{some other little bit}}$$

$$\frac{8 + .002}{+ .0003} \approx \frac{8.002}{\frac{3}{10000}} \approx \frac{80,020}{3}$$



$$\frac{8.002}{- .000084} \approx - (\text{large number})$$

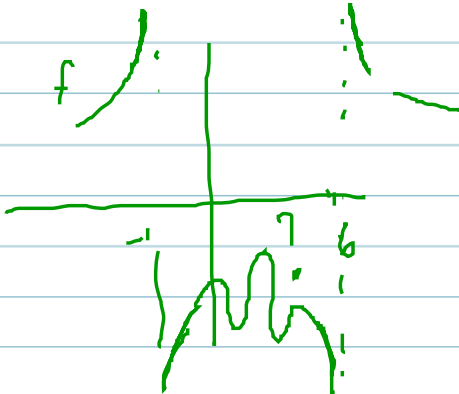
$x = -1$ is a vertical asymptote
asymptote

A vertical line the graph of f
can never cross, but the
graph gets closer & closer to that
line as x gets closer & closer
to -1

$$g(x) = x^2 - 5x - 6$$



$$g(-1 + \text{a little bit}) < 0$$



$$f(-1 - \text{a little bit}) \sim \frac{8}{+ \text{some little bit}} \gg 1$$